Exam. Code : 211004
Subject Code : 4991

M.Sc. (Mathematics) 4th Semester

DISCRETE MATHEMATICS–I

Paper—MATH-575

Time Allowed—2 Hours] [Maximum Marks—100

Note :— There are **EIGHT** questions of equal marks. Candidates are required to attempt any **FOUR** questions.

1. (a) Define partial ordered set and totally ordered set. What are the differences between them ? Give two examples of each.

   (b) Prove that distinct equivalence classes of an equivalence relation on a set form a partition of the set.

2. (a) State and prove extended form of pigeonhole principle. Give an example of it.

   (b) Find the number of the positive integers from 1 to 500 which are divisible by at least one of 3, 5 and 7.

3. (a) Define Conditional and Biconditional operators. Give three examples of both each.

   (b) Prove that $(p \wedge q) \rightarrow (p \wedge q)$ is tautology but $(p \vee q) \rightarrow (p \wedge q)$ is not.

4. (a) Prove that :

$$(p \rightarrow r) \vee (q \rightarrow s) = (p \wedge q) \rightarrow (r \vee s)$$

   (b) Construct the truth table for $(P \rightarrow Q) \wedge (Q \rightarrow P)$.

5. State and prove fundamental theorem of semi group homomorphism.

6. Define monoids with examples. Let $(G, *)$ and (G', o) be monoids with identities e and i respectively. Let $f : G \rightarrow G'$ be a homomorphism from $(G, *)$ onto (G', o) then $f(e) = i$.

7. (a) By finding the generating function of sequence S(n), find solution of recurrence relation.

   $S(n) - 2S(n-1) - 3S(n-2) = 0$, for $n \geq 2$, given $S(0) = 3$, $S(1) = 1$.

   (b) Define the Fibonacci sequence and find its closed form expression.

8. (a) Solve the recurrence relation :

   $S(k) - 7S(k-2) + 6S(k-3) = 0$, $S(0) = 8$, $S(1) = 6$, $S(2) = 22$

   (b) Solve the recurrence relation

$$S(k) + 5S(k-1) + 6S(k-2) = f(K),$$

   where $f(K) = \begin{cases} 0, & k = 0, 1, 5 \\ 6, & \text{otherwise} \end{cases}$, given that $S(0) = S(1) = 2$.